Annuitized Income and Optimal Asset Allocation

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Abstract

An investor who either buys an income annuity at retirement, or who has a higher level of guaranteed income through a pension or Social Security, should hold a different asset allocation than an investor who holds little guaranteed income. We use current annuity and bond prices to estimate optimal equity allocation for retirees with varying levels of guaranteed income who have higher and lower preference for income stability and bequests. We find that increasing annuitized income has a strong impact on optimal equity allocation. The average retiree will see their optimal equity allocation increase by roughly one percentage point for each percentage point increase in annuitized total wealth. Our results provide insight into prudent asset allocation recommendations for clients who haver higher levels of annuitized income.

Buying an income annuity with a portion of a retiree's retirement portfolio, or partial annuitization, efficiently reduces the risk of outliving safe investments when funding less flexible spending goals. The non-annuitized portfolio can then be used to achieve an equity risk premium when funding more flexible spending goals. Investment assets can also be used to maintain liquidity for uncertain spending needs or bequests.

After placing a portion of a retirement portfolio into an income annuity, how should an advisor prudently invest the remaining portfolio? Since a fixed immediate (or deferred) income annuity is constructed by an insurance company using a portfolio of bonds, intuition would suggest that the annuity should be purchased from the bond, or safe asset, portion of a portfolio. Unfortunately, there are is no established best practice for asset allocation when an advisor partially annuitizes a retirement portfolio.

A simple strategy of buying the annuity with bonds would result in an investment portfolio that appears riskier. For example, if an advisor buys a $125,000 income annuity from a $1 million portfolio with a 50/50 allocation to bonds and stocks, the remainder of the portfolio would consist of $375,000 in bonds and $500,000 in stocks (or a 59% equity allocation). But is this naïve partial annuitization portfolio reallocation to 59% equities as safe as a 50% equity allocation within a non-annuitized, investments-only portfolio if the goal is funding retirement income?

Our results show that the substitution of an annuity for bonds will reduce the income risk of the remaining portfolio, even thought that portfolio has a higher allocation to equities. Higher levels of guaranteed income generally result in riskier optimal client portfolios, consistent with the bond-like nature of guaranteed income; however, the actual portfolio impact varies considerably based on the situation and preferences of the investor.

Partially Annuitized Portfolios

Household portfolios are considered optimal when the allocation of risky assets provides just enough variance in future consumption to be worth the benefit from a higher expected return. Modern portfolio theory considers a two-asset portfolio of diversified risky assets and risk-free investments (Markowitz, 1991). The optimal mix of risky assets depends solely on a client's risk aversion. Clients with a higher coefficient of relative risk aversion prefer to hold a higher percentage of their portfolio in safe assets, and a more risk-tolerant investor will hold a higher allocation of risky assets.

More risk results in higher portfolio variance. Higher variance in the payout on investment assets leads to greater volatility in spending. Risk-averse investors prefer greater consumption certainty and are willing to forego a higher expected portfolio return in order to reduce variation in future spending. Investors and their advisors select the allocation that provides the highest expected utility over time.

This assumes that the client and advisor are investing for spending goals that have a defined time horizon. Retirement complicates the investment portfolio decision because the time horizon is unknown. Consumption volatility can occur as a result of both investment volatility and variance in longevity, or the risk of outliving assets. An unknown lifespan presents a third

dimension of consumption risk that favors investments that have a higher payout when clients live a long time.

Davidoff, Brown and Diamond (2005) show that an investor who wishes to fund income in retirement would maximize their utility by placing a healthy percentage of their savings in an annuity, even if they have some bequest motive. Running out of money could lead to a big decrease in spending, resulting in a significant reduction in welfare (utility). Milevsky, Moore and Young (2006) refer to this risk as the probability of financial ruin that occurs when the retiree runs out of money. Annuities reduce the utility consequence of running out of money by providing a guaranteed lifetime income that provides a higher level of minimum spending.

Substituting Income Annuities for Bonds

Annuities don't just cushion the blow of financial ruin. Many advisors work with clients to establish an acceptable failure rate from a retirement withdrawal strategy. For example, a client and advisor may decide that a 5% failure rate is acceptable. All else equal, a retiree who invests a dollar in an income annuity, rather than in a bond whose duration is matched to a safe spending need in retirement, will be able to spend more each year at the same failure rate because of the mortality credits provided by the annuity.

Pfau (2015) refers to income annuities as "actuarial bonds." Annuities can be viewed as a bond investment pooled with other retirees (the actuarial part) that provides protection against the risk of outliving assets unavailable with a non-annuitized bond investment. Pfau argues that an efficient retirement portfolio strategy involves substituting some, if not all, of a retiree's bond investments for income annuities.

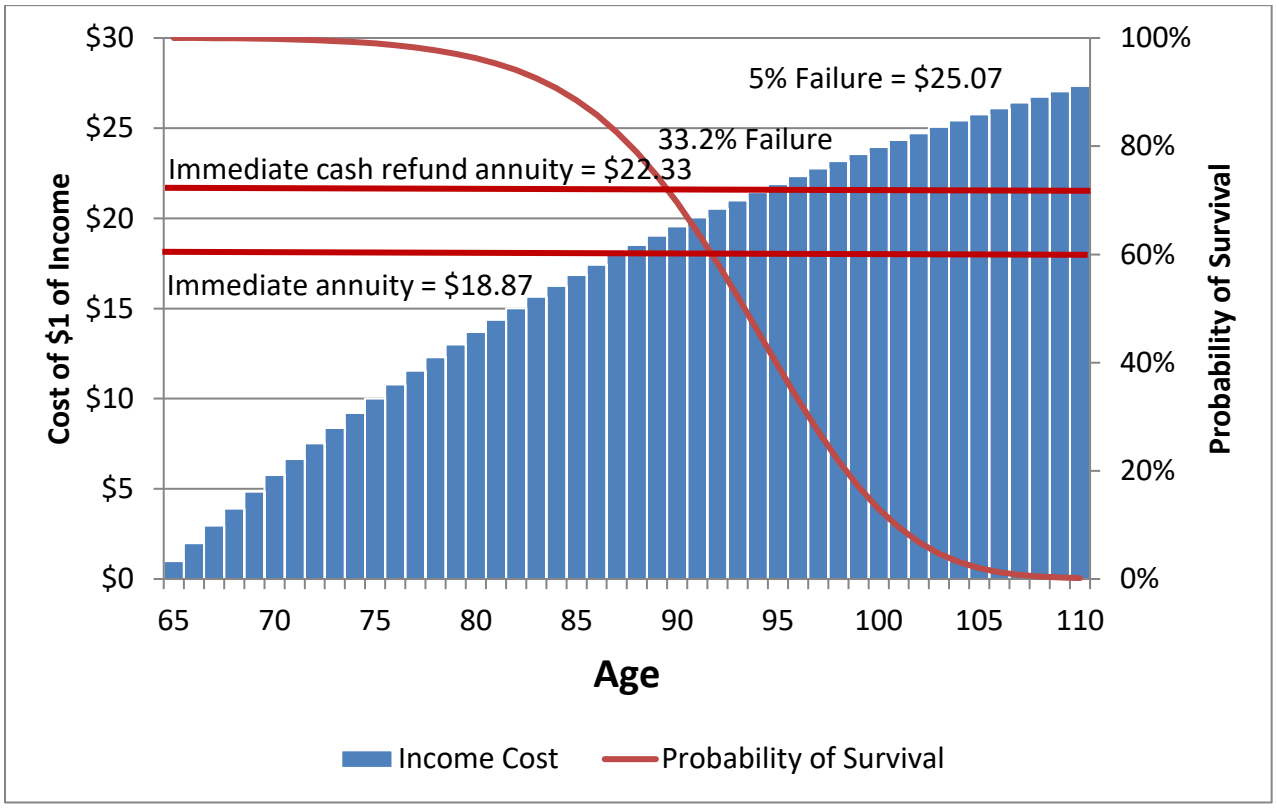
When asset returns are known at retirement, for example if the retiree creates a bond ladder from existing savings, the likelihood of ruin is affected by the probability that one will live longer than the last bond payment. Bond ladders provide a useful lens through which to also view the benefit of annuitization, since annuities will provide a higher expected income per dollar spent if the retiree lives longer than the average predicted by insurance company actuaries (assuming expenses on income annuities are modest, which they appear to be (Blanchett, 2016)).

A retiree who builds a bond ladder is exposed to greater risk of ruin and a lower expected income if they build the ladder to last beyond expected longevity. The longer the length of the bond ladder, say to age 95 or 100, the lower the probability of ruin but the higher the expected income gain from annuitization.

Annuitization allows the retiree to spend as if they have a time horizon equal to slightly longer than the average longevity (how much longer is determined by insurance costs). This means that they will spend more than if they had built a bond ladder well beyond the average longevity. Not only will they spend more than a ladder of bonds while the ladder is providing income, but they will spend much more if they outlive the final bond ladder payment.

Figure 1 illustrates the potential benefit of annuitization. We use the Society of Actuaries individual annuity mortality table[1] to illustrate the distribution of ages at which the surviving spouse is predicted to die, where we assume independence between spousal mortality rates. To estimate the cost of creating a bond ladder to provide income for various periods, we use the yield curve on nominal Treasury bonds[2] as of September 1, 2017. Annuity prices are estimated by taking the average of the top 5 quotes for a $100,000 joint and survivor immediate income annuity quotes from CANNEX. The annuity quotes are for a life only annuity and one that includes a cash refund provision. A cash refund provision ensures that the annuitant will at least receive the $100,000 paid for the annuity no matter what age of death.

Figure 1: A Comparison of Bond Ladders and Immediate Annuitization



If a retiree were to create a $100,000 bond ladder that lasted long enough to fund spending for 95% of couples, it would cost $25.07 per dollar of income. If they purchased an income annuity at today's rates, they would pay $18.87 per dollar of income and have a 100% chance of funding spending over the couple's lifetime. For a retiree concerned about not living long enough to get their money back on the annuity purchase, a joint and survivor annuity costs $22.33 per dollar of income. Had the retiree instead invested $22.33 in a bond ladder, the couple would have a

[1] http://www.naic.org/store/free/MDL-821.pdf

[2] https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

33.2% chance of outliving their savings. Buying an income annuity instead of a bond ladder allows a retiree to pay less for a safer income.

If the optimal retirement portfolio involves substituting some percentage of bonds with income annuities, what should the rest of the portfolio look like? As noted, the substitution of annuities for bonds provides a reduced likelihood of ruin if markets don't perform well and/or the retiree lives a long time.

This suggests that the rest of retiree's portfolio should, in fact, be more heavily weighted toward equities in order to provide the same amount of consumption risk as a non-annuitized portfolio. If a 60% bond/40% equity portfolio was appropriate for a moderately risk-averse retiree, what would happen if they invested half of their bond portfolio in an income annuity? Their total financial asset portfolio would now be 30% annuities, 30% bonds, and 40% equities. Their balance sheet, however, would now show a 57% equity allocation rather than a 40% equity allocation. Ironically, their 57% equity portfolio would appear riskier when, theoretically, it would pose less retirement income risk than a non-annuitized 40% equity portfolio.

Annuities in the Balance Sheet

In order to assess the portfolio consequences of buying an income annuity with investment assets, it is important to understand that retirees already have considerable annuitized wealth. According to ICI (2016), $10 trillion of the total $25 trillion in U.S. retirement assets is in defined benefit plans and annuities. This figure does not include the value of government pension benefits, such as Social Security retirement benefits, which we estimate to be approximately $8 trillion alone for retirees currently in payout status. The value of Social Security benefits for individuals not yet receiving benefits would likely more than double the total estimated value. Therefore, at least half of total assets that are going to be used to fund retirement are already in some form of guaranteed income.

To determine the value of a stream of guaranteed income (V_t), we must estimate the mortality-weighted net present value of future guaranteed income retirement benefits (GI_t). There are a variety of key assumptions that are important when running these estimates, such as the benefit payment amount, payment duration, whether or not there is a cost of living adjustment ($COLA_t$), the riskiness of the underlying cash flows, and mortality assumptions.

$$V_t = \sum_n^D \frac{q_{D-n} GI_t (1+COLA_t)^{D-n}}{(1+r_t)^{D-n}} \qquad [1]$$

The discount rate for any kind of net present value calculation should reflect the riskiness of the cash flows, where safer cash flows would use a lower discount rate. For example, when estimating the value of Social Security retirement benefits Treasury bonds would likely be a suitable discount rate. While counterparty risk exists for annuities, insurer's reserves and state guarantees suggest a discount rate of corporate bonds at a duration that matches the cash flow duration of the annuity. The longer duration of deferred annuities would require a higher discount rate assuming a positive term premium.

We provide some calculations to serve as guidance on how much a stream of guaranteed income is worth. The analysis uses the Treasury yield curve for discount rates and the Society of

Actuaries 2012 immediate annuity mortality table for mortality rates. Ideally, the discount rate should vary by the safety of guarantor of the guaranteed income and mortality should vary based on the investor, so these calculations are imperfect estimates.

The analysis assumes three household scenarios: single male, single female, or a joint couple (male and female both the same age). For the joint benefit, we assume payments continue, and don't decrease, when the first spouse passes away (which is called a 100% survivor benefit). This is consistent with how private pensions and annuities with a 100% spousal continuation benefit tend to work, but is different from Social Security retirement benefits since the surviving spouse typically receives the larger of the two individual benefits.

The numbers shown in Table 1 allow an advisor to estimate the approximate portfolio value of a guaranteed income source. For example, a married couple (male and female) both age 65 with a pension benefit that increases annually with inflation and starts immediately would have a multiplier of 25.7. If the expected income benefit was $50,000 (combined), the portfolio value of this benefit would be $1.285 million.

Table 1: Guaranteed Income Multipliers

Benefit Increases Annually with Inflation

Current Age	Male/Female Couple Same Age					Single Male					Single Female				
	Years till Benefit Starts					Years till Benefit Starts					Years till Benefit Starts				
	0	5	10	15	20	0	5	10	15	20	0	5	10	15	20
45	41.3	36.3	31.4	26.7	22.3	35.3	30.4	25.8	21.4	17.5	37.3	32.4	27.6	23.1	19.0
50	37.5	32.5	27.6	23.0	18.7	31.5	26.8	22.3	18.1	14.3	33.4	28.6	23.9	19.6	15.7
55	33.6	28.7	23.9	19.4	15.2	27.7	23.1	18.8	14.8	11.2	29.6	24.8	20.4	16.2	12.5
60	29.6	24.8	20.1	15.8	11.8	24.0	19.5	15.4	11.6	8.4	25.7	21.2	16.9	13.0	9.5
65	25.7	20.9	16.4	12.3	8.8	20.3	16.1	12.1	8.7	5.9	22.0	17.6	13.5	9.8	6.8
70	21.7	17.1	12.8	9.1	6.3	16.8	12.7	9.1	6.2	4.0	18.3	14.1	10.3	7.1	4.7
75	17.8	13.4	9.6	6.5	4.3	13.3	9.5	6.5	4.2	2.6	14.7	10.7	7.4	5.0	3.1
80	14.0	10.0	6.9	4.5	2.9	10.1	6.8	4.4	2.8	1.8	11.3	7.8	5.2	3.3	2.0
85	10.5	7.1	4.7	3.1	2.2	7.4	4.5	2.9	1.9	1.3	8.3	5.4	3.5	2.2	1.4

Benefit Stays the Same Every Year

Current Age	Male/Female Couple Same Age					Single Male					Single Female				
	Years till Benefit Starts					Years till Benefit Starts					Years till Benefit Starts				
	0	5	10	15	20	0	5	10	15	20	0	5	10	15	20
45	30.1	25.2	20.7	16.7	13.2	26.6	21.9	17.6	13.8	10.7	27.8	23.0	18.6	14.7	11.4
50	28.1	23.3	18.8	14.9	11.5	24.5	19.8	15.7	12.0	9.0	25.7	20.9	16.6	12.9	9.7
55	26.0	21.2	16.8	12.9	9.6	22.2	17.7	13.6	10.2	7.3	23.4	18.7	14.6	11.0	8.0
60	23.7	18.9	14.6	10.9	7.7	19.8	15.4	11.5	8.2	5.6	21.0	16.5	12.5	9.1	6.2
65	21.2	16.5	12.3	8.7	5.9	17.3	13.0	9.4	6.3	4.1	18.5	14.1	10.3	7.1	4.6
70	18.5	13.9	9.9	6.7	4.3	14.7	10.6	7.2	4.6	2.8	15.8	11.6	8.1	5.2	3.3
75	15.6	11.2	7.6	4.9	3.0	11.9	8.1	5.3	3.2	1.9	13.1	9.1	6.0	3.7	2.2
80	12.6	8.6	5.6	3.5	2.1	9.3	5.9	3.6	2.1	1.3	10.3	6.7	4.3	2.5	1.5
85	9.7	6.3	3.9	2.4	1.6	6.9	4.0	2.4	1.4	1.0	7.8	4.8	2.9	1.7	1.0

Estimating the current value of existing guaranteed income sources is an important first step needed to estimate the portfolio impact of purchasing additional annuitized income. For many households, the value of existing guaranteed income sources is significant – Social Security alone is a benefit that increases annually with inflation that is worth $500,000 or more for many couples.

Including the present value of annuitized income as part of a holistic balance sheet allows an advisor to better understand the resources that can be used to meet an income goal in retirement. Just because a mutual fund investment value appears on a quarterly statement does not make the asset any more valuable if the client's goal is to provide income in retirement. Similarly, when a retiree purchases an annuity this does not mean that their ability to meet their goals has been

reduced by the decrease in the value of their investible assets. Assets only have value because they can be used to meet the future spending and legacy needs of clients.

Optimal Equity Allocation and Guaranteed Income

In order to understand how holding annuitized wealth affects the safety of a retirement plan, we need to conduct an analysis to determine how the optimal equity allocation for a retiree household should change for a variety of situations and preferences. The base case scenario for the analysis is a couple, male and female, both age 65. While there are obviously considerable differences in the composition and ages of actual retiree households, the use of a single household type allows us to more easily evaluate the impact of adjusting different model parameters.

Mortality rates are based on the Society of Actuaries 2012 Immediate Annuity Mortality table (NAIC, 2016). This mortality table is used instead of the Social Security Administration's 2013 Period Life Table (SSA, 2016) because individuals who receive financial planning services tend to be wealthier than the average investor (Martin and Finke, 2014). This SOA table is more applicable to individuals (and households) with higher incomes since they have longer life expectancies, on average. For example, Chetty et al. (2016) note that the life expectancy of a 65-year-old in the top household income quintile is approximately three years longer than the median life expectancy. This is the same approximate increase in life expectancies when comparing the Society of Actuaries 2012 Immediate Annuity Mortality table to the Social Security Administration's 2013 Period Life Table for a 65-year-old.

Instead of modeling a single fixed retirement period, for example 30 years, the retirement model weights the probability of the retiree household surviving to each age. As noted by Blanchett and Blanchett (2008) and others, true success is a portfolio providing income for the life (or lives) of the retiree household and not over some arbitrary fixed period. The approach is discussed in additional detail in the utility model section. To provide some perspective on the distribution of mortality in the simulations, there is a 43% chance that at least one member of the couple (or both) will survive 30 years in retirement, a 16% probability of surviving 35 years, and a 3% probability of surviving 40 years.

The guaranteed income benefit for the analysis is assumed to increase annually with inflation for the entire period of the projection. This is slightly different than how actual Social Security retirement benefits work, where the surviving spouse receives the larger of the two benefits upon the death of the first spouse (not both benefits). The approach used for this model would be most consistent with an annuity that includes a cost-of-living adjustment tied to inflation and a 100% survivor continuation benefit upon the death of the first spouse.

The retirement income goal is assumed to be a combination of nondiscretionary and discretionary income goals (50% each). For the nondiscretionary portion of the income goal, the annual income need is assumed to increase each year based on inflation. This withdrawal amount is fixed regardless of the ongoing sustainability of the withdrawal amount. In other words, even if the portfolio is headed for certain ruin, that amount would still be withdrawn. For the discretionary portion of the income goal, the annual withdrawal is determined so that the funded ratio for the retiree is constant throughout retirement. The model used to estimate the dynamic withdrawal amount is identical to the approach introduced in Blanchett, Finke, and Pfau (2016).

A portfolio fee of 50 basis points is included to reflect modest investment management expenses. While the assumed fee is higher than the costs associated with investing in low-cost index mutual funds (which can be as low as about 5 bps today), it is lower than the fees associated with actively managed mutual funds (which can easily exceed 100 bps) and/or advisor asset management fees.

Each scenario test is based on a 1,000-run Monte Carlo projection where each run is assumed to last a maximum of 50 years (age 115 for the base case scenario). For each projection, initial withdrawal rates from 1.0% to 10.0% are considered in 0.1% increments (giving 91 test withdrawal rates). Taxes and required minimum distributions (RMDs) are ignored for simplicity. The following subsections detail additional key assumptions for the model.

Utility Model

According to economic theory, spending about the same amount each year in retirement provides the greatest satisfaction. A retiree's willingness to be flexile in terms of annual spending is conceptualized through the slope of the utility function, also known as the coefficient of relative risk aversion (Finke, Pfau and Williams, 2012). While there are several utility functions used to estimate risk aversion, the most common is a Constant Relative Risk Aversion (CRRA) utility function, shown in equation 2, where the amount of utility (U) received varies depending on level of consumption (c) and level of investor risk aversion (γ).

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad [2]$$

Implied within the CRRA utility function is the law of diminishing marginal utility in which losses (especially extreme losses) are weighted more heavily than gains. This function lends itself to retirement income modeling since it heavily penalizes scenarios where the retiree is left destitute.[3] A utility model is especially useful for retirement income modeling because it can incorporate a variety of preferences, such as a retiree's desire to have a more stable lifetime income.

The utility model used for this analysis is similar to the model introduced by Blanchett and Kaplan (2013), where the risk tolerance parameter and the elasticity of intertemporal substitution (EOIS) parameter are treated as separate. This is a recursive utility approach similar to the model introduced by Epstein and Zin (1989). For each simulated income path, the utility-equivalent constant income level is calculated using equation 2 where: I_t is the total income in year t; q_t is the probability of surviving to at least year t, based on data from the Society of Actuaries 2012 Immediate Annuity Period Mortality Table; T is the length of the simulation, which is equal to 50 when t = 1 (i.e., the simulation lasts to age 115 based on the initial ages of 65); ρ is the investor's subjective real discount rate, which is assumed to be 2%; and η is the elasticity of

[3] A minimum level of guaranteed income is always included to eliminate the possibility of an infinite marginal utility at zero income.

intertemporal substitution parameter, which is assumed to be equal to 0.5, 0.25, or 0.125 to reflect investors with low, moderate, or high levels of income stability preference, respectively.

$$II = \left(\frac{\sum_{t=0}^{T} q_t (1+\rho)^{-t} I_t^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{T} q_t (1+\rho)^{-t}} \right)^{\frac{\eta}{\eta-1}} \qquad [3]$$

The expected utility is measured by equation 4, where M is the number of paths, the subscript i denotes which of M paths is being referred to, p_i is the probability of path i occurring which is set to 1/ M, and θ is the risk tolerance parameter assumed to be 0.333.

$$EU = \sum_{i=1}^{M} p_i \frac{\theta}{\theta-1} (II_i)^{\frac{\theta-1}{\theta}} \qquad [4]$$

The optimal initial withdrawal rate maximizes the certainty-equivalent of the stochastic utility-adjusted income (Y) as noted in equation 5.

$$Y = \left[\sum_{i=1}^{M} p_i \, II_i^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \qquad [5]$$

Residual assets (bequests) are ignored in this model (i.e., the retiree household is assumed to care about maximizing retirement income.

Return Assumptions

The analysis uses two sets of returns, historical and forecasted. Historical returns are based on the approximate long-term average returns of U.S. stocks and bonds using the Ibbotson® SBBI® time series data (i.e., since 1926). The forecasted returns are based on Morningstar Investment Management LLC's 2016 capital market assumptions. These forecasted returns are time-varying, where returns in early retirement are assumed to be relatively low to reflect current market conditions, but are assumed to drift back toward the long-term average later in retirement. The historical returns are not assumed to be time-varying (they are constant over the entire projection). For simplicity purposes, the standard deviations and correlation values are the same for both sets of assumptions. There is no assumed serial correlation of returns. The assumptions for the respective series are shown in Table 2.

Table 2: Return Assumptions

Historical Return Assumptions					
	Return Period (Year)				
	1-10	11-20	21+	Std Dev	Correl
Stock	12.0%	12.0%	12.0%	20.0%	0.10
Bond	5.5%	5.5%	5.5%	6.0%	
Inflation	3.0%				

Future Return Assumptions					
	Return Period (Year)				
	1-10	11-20	21+	Std Dev	Correl
Stock	5.6%	7.5%	9.4%	20.0%	0.10
Bond	2.9%	3.6%	5.5%	6.0%	
Inflation	2.2%				

Equity Allocation Glide Path

The equity allocation (e) for the first year of retirement (t) is set at either 10%, 40%, or 70%. The equity allocation is assumed to decline each year during retirement consistent with the rate of change in the equity allocations for the Morningstar Lifetime Indexes.[4] Despite recent debate about the efficacy of declining glide paths during retirement, this remains the most common shape among retirement income products today. Equation 6 is used to determine the equity allocation, with annual rebalancing back to the target allocation.

$$e_t = (3.0962E\text{-}05(t-1)^2 + 3.9338E\text{-}04(t-1) + 0.975)e_{t-1} \qquad [6]$$

This approach treats the risk aversion associated with the portfolio as separate from the risk aversion associated with income. While the two may likely be related for some investors, other retirees will be willing to take substantial risk in their portfolio but may be far more risk averse with respect to funding retirement. This model allows for these varying preferences.

Results

Nine base cases are provided with three income stability preferences and three bequest preferences. Within each case there are 25 scenarios that vary by initial withdrawal rate and the percentage of that household's wealth is held in annuitized income. Retirement wealth is assumed to consist of two assets: annuitized income and the portfolio. Each dollar of annuitized income has a mortality-weighted net present multiple of 24.23, which is the present value of $1 of inflation-adjusted income for life weighted by mortality. For example, if a retiree household expected $50,000 in annual inflation-adjusted guaranteed income, the value of the annuitized income would be $1,211,444 ($50,000 x 24.23 = $1,211,444). If that household had retirement savings of $500,000, then the total value of the annuitized income would be 70.78% of the total

[4] https://corporate.morningstar.com/us/documents/Indexes/AssetAllocationsSummary.pdf

wealth for that household ($1,211,444/$1,711,444 = 70.78%). Other assets that could potentially be used to fund retirement, most notably home equity, are ignored in this analysis. Equity allocation represents the percentage of invested, non-annuitized wealth that should optimally be held in equity investments.

Table 4: Optimal Equity Allocation by Withdrawal Rate and Wealth in Guaranteed Income

Low Income Preference, Low Bequest Preference

Initial w%	Annuitized % of Wealth				
	5%	25%	50%	75%	95%
2%	15	15	15	15	15
3%	25	25	25	25	25
4%	35	35	35	40	40
5%	45	50	55	55	60
6%	60	60	70	75	75

Low Income Preference, Moderate Bequest Preference

Initial w%	Annuitized % of Wealth				
	5%	25%	50%	75%	95%
2%	70	95	100	100	100
3%	45	80	100	100	100
4%	40	60	95	100	100
5%	45	60	90	100	100
6%	60	70	90	100	100

Low Income Preference, High Bequest Preference

Initial w%	Annuitized % of Wealth				
	5%	25%	50%	75%	95%
2%	65	85	100	100	100
3%	45	75	100	100	100
4%	40	65	100	100	100
5%	45	65	100	100	100
6%	50	75	100	100	100

Moderate Income Preference, Low Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	15	15	15	15	15
3%	25	25	25	25	25
4%	35	35	35	35	40
5%	45	45	50	55	55
6%	50	60	65	70	75

Moderate Income Preference, Moderate Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	40	60	80	100	100
3%	25	40	65	100	100
4%	30	35	50	95	100
5%	40	45	55	90	100
6%	50	55	70	90	100

Moderate Income Preference, High Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	35	50	65	100	100
3%	25	40	60	100	100
4%	30	35	55	95	100
5%	35	45	55	95	100
6%	40	55	70	100	100

High Income Preference, Low Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	20	15	15	15	15
3%	25	25	25	25	25
4%	35	35	35	35	40
5%	45	45	50	50	55
6%	50	60	60	65	75

High Income Preference, Moderate Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	20	35	50	80	100
3%	20	25	35	70	100
4%	25	30	35	55	100
5%	30	40	50	55	100
6%	35	55	55	70	100

High Income Preference, High Bequest Preference

Initial w%	Annuitized % of Wealth				
	5	25	50	75	95
2%	20	30	40	65	100
3%	20	20	35	60	100
4%	20	30	35	55	100
5%	20	40	45	60	100
6%	25	50	55	70	100

The optimal equity allocation tends to increase at higher levels of guaranteed income, in some cases dramatically. For example, an investor with a moderate income and bequest preference using a 4% withdrawal rate, the optimal equity allocation with 5% of wealth in guaranteed income is 30% while the optimal equity allocation with 75% of wealth in guaranteed income is 95%. Those who dislike income volatility, or who have a high preference for stable income, see the most modest increases in equity allocation by guaranteed income – mainly because they strongly dislike income volatility and have a low optimal equity allocation in general. Interestingly, when these same investors have a high enough percentage of annuitized income they are willing to accept significant investment risk if they have a bequest preference.

These results suggest that when a client shifts a portfolio from investment assets into guaranteed income, the remainder of the portfolio should optimally contain a significantly higher allocation to risky assets. For example, a client with a moderate risk aversion, 4% withdrawal rate, and moderate bequest preference may hold $500,000 of wealth in Social Security and $1.5 million in investable assets. They will optimally hold 35% of investment assets in equity, or $525,000. If they were to purchase a $500,000 inflation-adjusted annuity with investment assets, the remaining $1 million portfolio would optimally consist of a 50% allocation to equities, or $500,000. This means that $475,000 of the annuity would optimally be purchased with fixed-income assets and $25,000 would be purchased from equity investments.

Annuity Purchase Portfolio Impact

While the previous analysis provides analysis on how the equity allocation of a portfolio should vary based on the percentage of guaranteed income, it does not provide more precise guidance for advisors about how much of the cost of an annuity should be paid from fixed income or equity investments.

Annuity quotes are obtained from CANNEX on January 6, 2017 for a heterosexual couple both age 65, with a 100% continuation benefit, a full cash refund and no cost of living adjustment. These riders/attributes are selected since they are the most features selected among investors requesting annuity quotes from CANNEX. We use the average payout rate for the five highest quotes. Quotes are obtained for income that commences at 65, 70, 75, 80, and 85, where the respective payout rates are 5.29%, 7.12%, 10.01%, 15.09%, and 23.52%. The payout rate is the amount of income the investor would receive for an annuity amount purchased. For example, a payout rate of 10.01% would mean that for every $100,000 used to purchase the annuity, the retiree would start receiving $10,010 a year at age 75 for as long as either member of that couple is living. For convenience, the analysis assumes a 4% initial withdrawal rate and 50% of retirement need is covered from annuitized income.

Table 5: Optimal Portfolio Allocations with an Annuity Purchase Where Income Begins at Age 75

Low Income Preference, Low Bequest Preference		
Stocks	Bonds	Annuity
$350	$650	$0
$425	$425	$150
$420	$280	$300
$330	$220	$450
$200	$200	$600

Low Income Preference, Moderate Bequest		
Stocks	Bonds	Annuity
$950	$50	$0
$850	$0	$150
$700	$0	$300
$550	$0	$450
$400	$0	$600

Low Income Preference, High Bequest Preference		
Stocks	Bonds	Annuity
$1,000	$0	$0
$850	$0	$150
$700	$0	$300
$550	$0	$450
$400	$0	$600

Moderate Income Preference,		
Stocks	Bonds	Annuity
$350	$650	$0
$383	$468	$150
$350	$350	$300
$303	$248	$450
$200	$200	$600

Moderate Income Preference,		
Stocks	Bonds	Annuity
$500	$500	$0
$510	$340	$150
$490	$210	$300
$468	$82	$450
$400	$0	$600

Moderate Income Preference,		
Stocks	Bonds	Annuity
$550	$450	$0
$553	$298	$150
$525	$175	$300
$495	$55	$450
$400	$0	$600

High Income Preference, Low Bequest Preference		
Stocks	Bonds	Annuity
$350	$650	$0
$340	$510	$150
$315	$385	$300
$303	$248	$450
$200	$200	$600

High Income Preference, Moderate Bequest		
Stocks	Bonds	Annuity
$350	$650	$0
$340	$510	$150
$350	$350	$300
$303	$248	$450
$280	$120	$600

High Income Preference, High Bequest Preference		
Stocks	Bonds	Annuity
$350	$650	$0
$340	$510	$150
$315	$385	$300
$303	$248	$450
$280	$120	$600

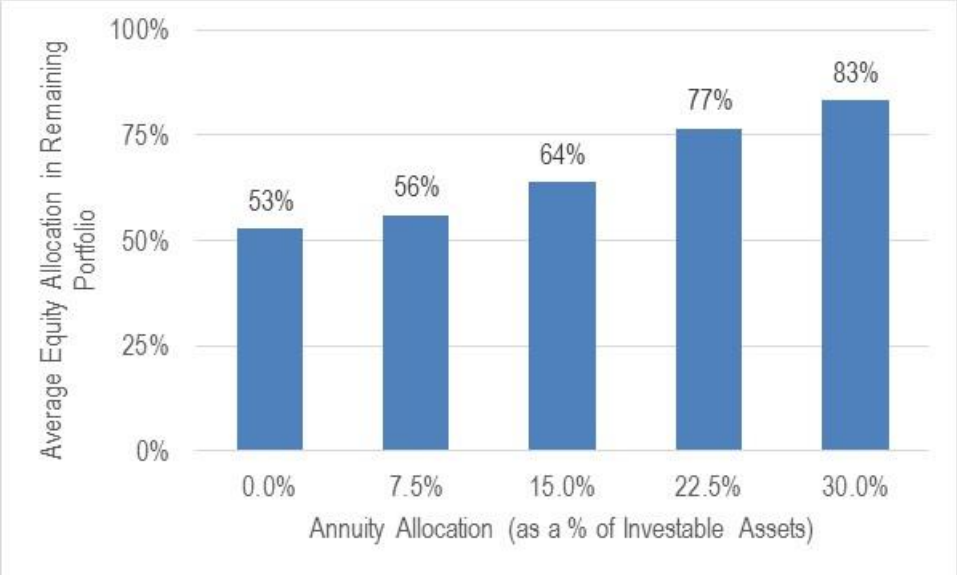
Results from Table 5 show the incremental increase in optimal equity allocation as investment assets are used to purchase an annuity. For a retiree with a moderate ability to withstand changes in income and a moderate bequest motive, an optimal $1 million investment portfolio consists of $500,000 in stocks and $500,000 in bonds when no annuity is purchased. Upon purchasing a

$150,000 annuity, the optimal portfolio is now $510,000 in stocks and $340,000 in bonds. Although moving from a 50% equity investment portfolio to a 60% equity portfolio may appear riskier, in reality this is the optimal amount of investment risk for a retiree with average preferences who has purchased annuitized income. Buying a $300,000 annuity moves the optimal allocation to $490,000 in equities and $210,000 in bonds, or a 70% equity allocation. The more annuitized income the retiree purchases, the higher the optimal percentage of stocks in their remaining investment portfolio.

Even a risk averse retiree who places a large value on income stability will, in the absence of a bequest motive, optimally hold an investment asset allocation that is a higher allocation of equities than bonds when their annuitized income represents 40% of total assets. For those risk averse retirees who have a moderate bequest motive, the optimal allocation to equities is 50% when annuitized income represents 30% of total wealth. Once a retiree has purchased a significant amount of annuitized income, even if they are risk averse, it is optimal to carry a stock allocation within their investment portfolio that appears risky. This increased volatility in their investment portfolio will have less of an impact on the risk of their overall income because they already have a larger base of stable guaranteed income to spend.

For further insight into the incremental change in optimal asset allocation when an annuity is purchased, Figure 2 shows the increase in allocation to equities that is optimal when a client purchased additional guaranteed income using the average of the 9 different client preferences in Table 5. Moving from no annuitized income to 15% of total assets increases optimal allocation from 53% to 64% on average. Moving to 30% increases optimal equity allocation to 83%. A higher percentage of wealth held in annuitized income means a higher optimal equity allocation for an average client.

Figure 2: Average Equity Allocation in Remaining Portfolio



Conclusion

Advisors who are considering a partial annuitization strategy in which a portion of retirement assets are allocated to an income annuity have received little guidance on how they should optimally invest the remainder of a portfolio. Using actual annuity price quotes and current bond returns, we estimate optimal equity allocations at varying levels of annuitized income among retirees with higher and lower income stability and bequest preferences.

Our results confirm the intuition that the when a retiree buys an income annuity, they should optimally take more investment risk with the remainder of their investment portfolio. Even risk-averse retirees should hold a significant allocation to equities if they have a large percentage of total wealth held in guaranteed income assets.

Our results have several important planning implications. For example, the U.S Department of the Treasury has created new rules to incentivize the adoption of deferred income annuities through qualified longevity annuity contracts. Retirees who purchase longevity insurance should optimally increase stock allocation in the remainder of their portfolio. Clients with employer pensions should optimally hold a higher allocation of their investment portfolio in equities, and even risk-averse retirees with significant pension assets and Social Security should likely hold most of their investment portfolio in equities. Conversely, average retirees with no guaranteed income outside of Social Security should optimally hold a lower percentage of their investment assets in equities.

Advisors considering the purchase of an income annuity should optimally draw from bond assets to maintain an appropriate equity allocation. Although the optimal percentage depends on a number of factors including bequest and stable income preference, we find that a good rule of thumb is that annuities should be purchased with about 80% of assets from bond investments and 20% from equities.

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